OCR
RECOGNISING ACHIEVEMENT

Wednesday 16 May 2012 – Morning

AS GCE MATHEMATICS

4721 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer book 4721
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

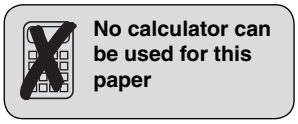
These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



DC (SJF/JG) 51089/2

1 Simplify $(x-5)(x^2+3)-(x+4)(x-1)$. **[3]**

2 Express each of the following in the form 7^k:

(i) $\sqrt[4]{7}$, **[1]**

(ii) $\frac{1}{7\sqrt{7}}$, **[2]**

(iii) $7^4 \times 49^{10}$. **[2]**

3 **(i)** Find the gradient of the line l which has equation $3x-5y-20=0$. **[1]**

(ii) The line l crosses the x-axis at P and the y-axis at Q. Find the coordinates of the mid-point of PQ. **[4]**

4 **(i)** Express $2x^2-20x+49$ in the form $p(x-q)^2+r$. **[4]**

(ii) State the coordinates of the vertex of the curve $y=2x^2-20x+49$. **[2]**

5 **(i)** Sketch the curve $y=\sqrt{x}$. **[2]**

(ii) Describe the transformation that transforms the curve $y=\sqrt{x}$ to the curve $y=\sqrt{x-4}$. **[2]**

(iii) The curve $y=\sqrt{x}$ is stretched by a scale factor of 5 parallel to the x-axis. State the equation of the transformed curve. **[2]**

6 Find the equation of the normal to the curve $y=\frac{6}{x^2}-5$ at the point on the curve where $x=2$. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. **[7]**

7 Solve the equation $x-6x^{\frac{1}{2}}+2=0$, giving your answers in the form $p\pm q\sqrt{r}$, where p, q and r are integers. **[6]**

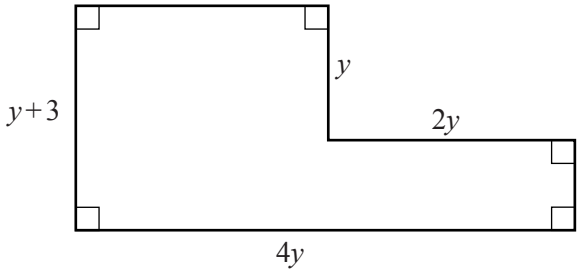
8 **(i)** Find the coordinates of the stationary point on the curve $y=x^4+32x$. **[5]**

(ii) Determine whether this stationary point is a maximum or a minimum. **[2]**

(iii) For what values of x does x^4+32x increase as x increases? **[1]**

9 **(i)** A rectangular tile has length $4x$ cm and width $(x + 3)$ cm. The area of the rectangle is less than 112 cm^2. By writing down and solving an inequality, determine the set of possible values of x. **[6]**

(ii) A second rectangular tile of length $4y$ cm and width $(y + 3)$ cm has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y. **[5]**

10 A circle has equation $(x - 5)^2 + (y + 2)^2 = 25$.

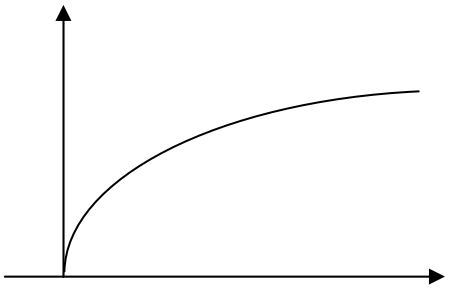
(i) Find the coordinates of the centre C and the length of the diameter. **[3]**

(ii) Find the equation of the line which passes through C and the point P (7, 2). **[4]**

(iii) Calculate the length of CP and hence determine whether P lies inside or outside the circle. **[3]**

(iv) Determine algebraically whether the line with equation $y = 2x$ meets the circle. **[5]**

Question			Answer	Marks	Guidance	
1			$x^3 - 5x^2 + 3x - 15 - (x^2 + 4x - x - 4)$	M1	Attempt to expand both pairs of brackets	No more than one "missing term"
				A1	Expansion with at most one incorrect term (no missing terms)	**Do not allow** "invisible brackets" unless final answer correct Allow one simplified incorrect term e.g. $(x^2 + 5x - 4)$
			$= x^3 - 6x^2 - 11$	A1 **[3]**	cao	
2	(i)		$\sqrt[4]{7} = 7^{\frac{1}{4}}$	B1 **[1]**	Allow $7^{0.25}$, $k = 0.25$ etc.	
2	(ii)		$\frac{1}{7\sqrt{7}} = 7^{-\frac{3}{2}}$	M1	**Clear evidence** of **correct use** of $7^a \times 7^b = 7^{a+b}$ or a single term $\frac{1}{7^d} = 7^{-d}$	Allow $\frac{1}{7^d 7^e} = (7^d 7^e)^{-1}$ [not $= 7^d 7^{-e}$]
				A1 **[2]**	Allow -1.5, $k = -1.5$ etc.	
2	(iii)		$7^4 \times 7^{20}$ $= 7^{24}$	M1 A1 **[2]**	7^{20} or 49^2 seen (or 49^{12}) Allow $k = 24$	$(7^2)^{10}$ is **not** good enough for M1
3	(i)		$\frac{3}{5}$	B1 **[1]**	Allow 0.6 or any equivalent fraction	Do not allow $\frac{3}{5}x$ as final answer
3	(ii)		$P\left(\frac{20}{3}, 0\right)$	B1	May be implied by subsequent working	Allow $x = \frac{20}{3}$ for P
			Q (0, -4)	B1	May be implied	Allow y = - 4 for Q
			$\left(\frac{\frac{20}{3}+0}{2}, \frac{0+^-4}{2}\right)$	M1	Correct method to find midpoint of line	Check formula, or if formula not seen, the use of formula is correct (including correct signs) for both x and y, Can be implied by correct final answers **SC**
			$\left(\frac{10}{3}, -2\right)$	A1 **[4]**	Allow exact equivalent forms, decimals must be correct to at least 2dp	If P and Q given the wrong way round but then used correctly to obtain correct final answer **B2**

Question			Answer	Marks	Guidance	
4	(i)		$2(x^2 - 10x) + 49$ $= 2(x-5)^2 - 50 + 49$ $= 2(x-5)^2 - 1$	B1 B1 M1 A1 **[4]**	$p = 2$ $(x-5)^2$ $49 - 2q^2$ or $\frac{49}{2} - q^2$	If p, q, r found correctly, then **ISW** slips in format. $2(x-5)^2 + 1$ **B1 B1 M0 A0** $2(x-5) - 1$ **B1 B1 M1 A1 (BOD)** $2(x-5x)^2 - 1$ **B1 B0 M1 A0** $2(x^2-5)^2 - 1$ **B1 B0 M1 A0** $2(x+5)^2 - 1$ **B1 B0 M1 A1 (BOD)** $2x(x-5)^2 - 1$ **B0 B1M1A1**
4	(ii)		(5, –1)	B1 FT B1 FT **[2]**	ft their q (Do not allow "5x") ft their r (Do not allow "-1y")	If restarted then **B1 B1** for each **B0** if more than one answer given
5	(i)			M1 A1 **[2]**	Correct shape of graph in Q1 Ignore reflection in the x axis Correct graph in Q1 only	Ignore "feathering" Finite "plot" scores **M0** Need not meet origin for **M** mark Allow slight curve downwards for **M** mark but not for **A** Allow tending to horizontal
5	(ii)		Translate(d) or Translation Parallel to x-axis, (+)4 units	B1 B1 **[2]**	**Do not accept** "shift", "move" etc. without the word translation/translate(d) For "parallel to the x axis" allow "horizontally", "across", "to the right", "in the (positive) x direction". **Do not accept** "in/on/across/up/along/to/towards the x axis"	Allow e.g. "4 units across in the positive x direction parallel to the x axis" but do not award second B1 if statements are contradictory. "Factor 4" not acceptable
5	(iii)		$y = \sqrt{\left(\frac{x}{5}\right)}$	M1 A1 **[2]**	$\sqrt{5x}$ or $\sqrt{\frac{x}{5}}$ seen **Must have** "$y =$ " to earn A mark (do not allow "$f(x) =$")	**SC** If doubt over whether use of square root/solidus is totally correct **B1** (Must still have "$y =$ ") Allow $\sqrt{5}y = \sqrt{x}$ or equivalent

Question	Answer	Marks	Guidance	
6	$\frac{dy}{dx} = -12x^{-3}$	M1 A1	Attempt to differentiate (i.e. kx^{-3} seen) Correct derivative	"+ C" is A0
	When $x = 2$, $\frac{dy}{dx} = -\frac{3}{2}$	A1	Correct value of $\frac{dy}{dx}$. Allow equivalent fractions.	
	Gradient of normal = $\frac{2}{3}$	B1 FT	Follow through their **evaluated** $\frac{dy}{dx}$	Must be processed correctly
	When $x = 2$, $y = -\frac{7}{2}$	B1	Correct y coordinate, accept equivalent forms	
	$y + \frac{7}{2} = \frac{2}{3}(x - 2)$	M1	Correct equation of straight line through (2, their evaluated y), any non-zero gradient	
	$4x - 6y - 29 = 0$	A1 **[7]**	Correct equation in required form i.e. $k(4x - 6y - 29) = 0$ for integer k. **Must have** "=0".	
7	$k = x^{\frac{1}{2}}$ $k^2 - 6k + 2 = 0$	M1*	Use a substitution to obtain a quadratic with k^2, $6k$ and 2(may be implied by squaring or rooting later)	**Any sight of 4 or 36x from "squaring" original equation scores 0/6.** Alternative solution: $6\sqrt{x} = x + 2$ $36x = x^2 + 4x + 4$ Rearrange and square both sides **M1***
	$(k - 3)^2 - 7 = 0$	M1 dep	Correct method to solve resulting quadratic	Correct simplified quadratic $x^2 - 32x + 4 = 0$ **A1** Method to solve quadratic **M1dep**
	$k = 3 \pm \sqrt{7}$	A1	$k = 3 \pm \sqrt{7}$ **or** $k = \frac{6 \pm \sqrt{28}}{2}$ **or** $k = 3 \pm \frac{\sqrt{28}}{2}$	Correct unsimplified expression **A1** Correct discriminant **A1** $16 \pm 6\sqrt{7}$ o.e. **A1**
	$x = (3 \pm \sqrt{7})^2$	M1 M1	Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)	**SC** If no evidence of substitution at start and no squaring/rooting at end: Correct method for solving quadratic with $a = 1$, $b = -6$, $c = 2$ and solution simplified to $3 \pm \sqrt{7}$ **B1**
	$x = 16 + 6\sqrt{7}$ or $x = 16 - 6\sqrt{7}$	A1 **[6]**	Allow $16 \pm 3\sqrt{28}$ or $16 \pm 2\sqrt{63}$	

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dy}{dx} = 4x^3 + 32$	M1 A1	Attempt to differentiate (one term correct) Completely correct	"+ C" is A0
			$4x^3 + 32 = 0$	M1	Sets their $\frac{dy}{dx} = 0$ (can be implied)	
			$x = -2$	A1	Correct value for x (**not** ±2) **www**	
			$y = -48$	A1 FT **[5]**	Correct value of y for *their* single non-zero value of x	e.g. (2, 80), (4, 384), (− 4, 128), (8, 4352), (− 8, 3840)
8	(ii)		$\frac{d^2y}{dx^2} = 12x^2$ When $x = -2$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	M1 A1 **[2]**	Correct method for determining nature of a stationary point – see right hand column Fully correct for $x = -2$ only	e.g. evaluating second derivate at x = "−2" and stating a conclusion Evaluating $\frac{dy}{dx}$ either side of x = "−2" Evaluating y either side of x = "−2"
8	(iii)		$x > -2$	B1 **FT** **[1]**	ft from single x value in **(i)** consistent with **(ii)**	**Do not accept** $x \geq -2$
9	(i)		Area of tile = $4x(x + 3)$	B1	Correct expression for area of rectangle (may be unsimplified)	
			$4x(x + 3) < 112$ $4x^2 + 12x - 112 < 0$	B1 √	Correct inequality for their expression	Correct alternative forms for factorised inequality include:
			$4(x + 7)(x - 4) < 0$	M1	Correct method to solve a three term quadratic	$(x + 7)(4x - 16) < 0$
				M1	Chooses correct region for the quadratic inequality i.e. lower root < x < higher root	$(4x + 28)(x - 4) < 0$ $(2x + 14)(2x - 8) < 0$ etc.
			$-7 < x < 4$	A1	(May be implied by correct final answer)	
			$\therefore\ 0 < x < 4$	A1 **[6]**	Restricts range to positive values of x **CWO**	**Do not allow ≤ for final A mark**
9	(ii)		Perimeter = $4y + (y + 3) + 2y + y + 2y + 3$	M1 A1	**Clear** attempt to add lengths of all **6** edges Correct perimeter simplified to $10y + 6$ seen	**Allow < or ≤ throughout part (ii)**
			$20 < 10y + 6 < 54$	B1 FT M1	Correct inequalities for their expression Solving 2 linear equations or inequalities dealing with all 3 terms	Can still be unsimplified here
			$1.4 < y < 4.8$	A1 **[5]**	Accept "$1.4 < y$, $y < 4.8$", "$1.4 < y$ ***and*** $y < 4.8$" but **NOT** "$1.4 < y$ ***or*** $y < 4.8$".	**Do not ISW** if contradictory incorrect form follows correct answer

Question			Answer	Marks	Guidance	
10	(i)		Centre (5, –2) Radius = 5 Diameter = 10	B1 M1 A1 **[3]**	5 or $\sqrt{25}$ soi	
10	(ii)		Gradient of line = $\frac{2-^-2}{7-5}(=2)$ $y-2=2(x-7)$ or y–$^-$2 = 2 (*x*–5) $y=2x-12$	M1 A1 M1 A1 **[4]**	uses $\frac{y_2-y_1}{x_2-x_1}$ with their centre correct equation of straight line through (7, 2) or their centre, any non-zero gradient o.e. 3 term equation	3/4 substitutions correct Allow other points on the line e.g. mid-point is (6,0)
10	(iii)		$\sqrt{(7-5)^2+(2-^-2)^2}$ $=\sqrt{20}$ $\sqrt{20}$ < 5 so *P* lies inside the circle	M1 A1 B1 FT **[3]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ with their centre Compares their length *CP* with their radius and states consistent conclusion. **Both lengths must be mentioned.**	3/4 substitutions correct**. Must have** square root as length specifically asked for. **SC** If **M0**, award for **B1** for finding $CP^2 = 20$ and stating 20 < 25 and concluding inside **www**
10	(iv)		$(x-5)^2+(2x+2)^2(=25)$ $(x-5)^2+(2x+2)^2=25$ $x^2-10x+25+4x^2+8x+4=25$ $5x^2-2x+4=0$ $b^2-4ac=4-(4\times5\times4)$ $b^2-4ac<0$ so no real roots	M1* A1 A1 M1dep A1 **[5]**	Substitute for *x*/*y* or attempt to eliminate one of the variables Correct unsimplified equation (= 0 can be implied) Obtain correct 3 term quadratic Attempt to determine whether equation has real roots with consistent conclusion regarding roots/intersection Fully justified statement that line and circle do not meet **www**	 If *x* eliminated, $5y^2-4y+16=0$ If the discriminant is evaluated, this must be –76 (from the quadratic in *x*) or –304 (from the quadratic in *y*) for full marks.